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# Second-law-based optimization of heat exchanger networks using load curves

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**Abstract**—The problem of simultaneously optimizing heat exchangers for a number of hot and cold fluids is approached from the viewpoint of the second law of thermodynamics. If the total duty requirements are fixed, i.e. all inlet and outlet temperatures are established for the fluids, then the resulting entropy production rate is independent of the actual pairings of the fluids. Thus, optimization with respect to size becomes essentially a maximization of the temperature differences between the paired fluids for all of the fluids considered.

Next it is shown that the same optimization can be obtained from the load curves where the temperature is plotted against the heat transfer for each of the fluids. If the film coefficient for any of the fluids is significantly different from the others, a simple shifting can be accomplished and a criterion is given to determine if such shifting improves the optimization. The method is extremely simple and can be performed most effectively on graph paper.

## 1. INTRODUCTION

DUE TO the rising costs of energy, the design of an optimal heat exchanger network between hot and cold process streams is an important problem. A heat exchanger network or train is defined as several heat exchangers arranged in series and parallel to effect heat exchange between several hot and cold streams. The objective of this work was to minimize the total area required for heat exchange using a thermodynamic approach. Other economic factors were not taken into consideration.

Early optimization work on heat exchanger networks was reviewed by Hendry *et al.* [1] and Raghavan [2]. Some of the early authors used heuristic approaches for stream matching which do not guarantee optimality. A practical method of evaluating the heat exchange among multiple streams in parallel and in series was developed by Chato *et al.* [3]. This can be used most effectively in the final design stage of a heat exchanger network. Lee *et al.* [4] used a mathematical approach based on a branch and bound method to synthesize networks and find the optimum This procedure requires too much time for large-scale problems.

Nishida *et al.* [5] used an algorithm which they claimed gave the least total heat transfer area and, by using a few heuristic rules, reduced it to a final form. Raghavan argued that this method did not guarantee optimality and proposed a *H*eat *A*vailability *F*unction (HAF) with which the maximum recoverable energy could be computed. An algorithm to determine an optimal network was also presented. In 1978, Linnhoff and Flower [6] presented a method similar to that presented by Raghavan in that they proposed a method which firstly generated preliminary networks which gave maximum heat recovery and then a final network was evolved using the preliminary networks as starting points. Malhotra *et al.* [7] used the Discrete Maximum Principle to minimize the total cost of heat exchanger chains in which one cold stream was heated by several hot auxiliary streams. In an extension to this work, Siddique *et al.* [8] considered the optimization of a heat exchanger train in which one cold stream was to be heated by several hot streams using the same procedure as in their earlier work. Recently Siddique *et al.* [9] extended their earlier work to include two cold streams. This method can only be applied to small-scale problems and is thus restrictive.

Parkinson *et al.* [10] considered the optimal design of resilient heat exchanger networks. A resilient network is defined as one which can tolerate fluctuations in stream temperatures and flow rates. They presented an algorithm which synthesized networks resilient for all stream fluctuations and minimized network investment costs for the conditions of maximum energy recoverable. In an extension of their earlier work, Parkinson *et al.* [11] also solved the problem of resilient heat exchanger network optimization using Monte Carlo simulation.

Hesselmann [12] presented an approach which incorporated the economic aspect of heat exchanger design. A functional relation between minimum investment costs and energy losses was determined and, by plotting the investment costs against energy losses, an optimal network could be determined.

The above approaches to the problem of heat exchanger network optimization rely on large computer-aided algorithms and are time consuming. A hand application technique has been used by Linnhoff and Turner [13] and Hindmarsh *et al.* [14] to optimize various process systems, including heat exchanger (or heat-recovery) systems. We present a simpler method based on the second law of thermodynamics, using

## NOMENCLATURE

| | |
|---|---|
| $A$ | heat transfer area in a fluid [m$^2$ or ft$^2$] |
| $A_f$ | fin area in a fluid [m$^2$ or ft$^2$] |
| $c_p$ | specific heat at constant volume [J kg$^{-1}$ K$^{-1}$ or Btu lb$^{-1}$ °F$^{-1}$] |
| $C$ | heat capacity rate, $\dot{m}c_p$ [W K$^{-1}$ or Btu h$^{-1}$ F$^{-1}$] |
| $CS$ | cold stream |
| $F$ | configuration factor for heat exchangers (= 1 for counterflow) |
| $h$ | heat transfer coefficient of a fluid [W m$^{-2}$ K$^{-1}$ or Btu h$^{-1}$ ft$^{-2}$ °F$^{-1}$] |
| $HS$ | hot stream |
| $\dot{I}$ | irreversibility production rate, $T_e\dot{S}$ [W or Btu h$^{-1}$] |
| $\dot{m}$ | mass flow rate [kg s$^{-1}$ or lbm h$^{-1}$] |
| $p$ | pressure [Pa or lbm h$^{-2}$ ft$^{-1}$] |
| $q$ | heat transfer rate [W or Btu h$^{-1}$] |
| $q_o$ | heat transfer rate in a heat exchanger [W or Btu h$^{-1}$] |
| $q_T$ | total heat transfer rate in a heat exchanger train [W or Btu h$^{-1}$] |
| $R$ | thermal resistance [K W$^{-1}$ or °F h Btu$^{-1}$] |
| $\dot{S}$ | entropy production rate [W K$^{-1}$ or Btu h$^{-1}$ °F$^{-1}$] |
| $T$ | absolute temperature [K or °F abs ($R$)] |
| $T_e$ | appropriately selected environmental temperature [K or °F abs ($R$)] |
| $v$ | specific volume [m$^3$ kg$^{-1}$ or ft$^3$ lbm$^{-1}$]. |

Greek symbols

| | |
|---|---|
| $\delta$ | temperature difference defined in Fig. A1 [K or °F] |
| $\Delta T$ | local temperature difference between hot and cold fluids [K or °F] |
| $\overline{\Delta T}$ | average temperature difference between hot and cold fluids, $(T_{hi}-T_{co}+T_{ho}-T_{ci})/2$ for counterflow [K or °F] |
| $\eta_f$ | fin efficiency |
| $\eta_o$ | overall surface efficiency for heat transfer, $1-(A_f/A)(1-\eta_f)$ |
| $\Sigma$ | defined by equation (7). |

Subscripts

| | |
|---|---|
| c | cold fluid |
| h | hot fluid |
| i | inlet |
| lm | log mean |
| o | outlet |
| p | due to pressure |
| w | wall |
| 1,2 | at the ends of a heat exchanger or heat exchanger number. |

the load curves for a particular problem. The method can easily be performed on a graph.

## 2. THERMODYNAMIC OPTIMIZATION OF THE HEAT EXCHANGE PROCESS

Thermodynamically, one form of optimization is the minimization of the entropy, $\dot{S}$, or the irreversibility, $\dot{I} = T_e\dot{S}$, production rates. For a single heat exchanger with only two process fluids, i.e. one hot and one cold stream, the net entropy production rate in the heat exchange process is

$$\dot{S} = \oint d\dot{S} = \int dq/T = \int_0^{q_o} \frac{dq_{hot}}{T} + \int_0^{q_o} \frac{dq_{cold}}{T}. \quad (1)$$

This set of integrals is represented by the net area between the curves $1/T$ vs $q$ for the two fluids as illustrated in Fig. 1.

Introducing the heat capacity rate

$$C \equiv \dot{m}c_p \quad (2)$$

we obtain

$$\dot{S} = \int_{T_{hi}}^{T_{ho}} \frac{C_h\, dT_h}{T_h} + \int_{T_{ci}}^{T_{co}} \frac{C_c\, dT_c}{T_c}. \quad (3)$$

If the heat capacity rates are constant, equation (3) can be integrated directly as

$$\dot{S} = C_c \ln (T_{co}/T_{ci}) - C_h \ln (T_{hi}/T_{ho}) > 0. \quad (4)$$

Thus, if the heat capacity rates and inlet and outlet temperatures (i.e. the total heat exchange or load, $q_o$) are fixed, the entropy production rate is also determined, regardless of the physical configuration of the heat exchanger. The same argument can be extended to multiple streams with more than one hot or cold

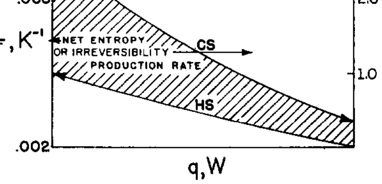


FIG. 1. Typical $1/T$ or $T_e/T$ vs $q$ curves for a two-fluid heat exchanger. The area under the cold stream curve, CS, is the rate of entropy or irreversibility increase, while the area under the hot stream curve, HS, is the rate of entropy or irreversibility decrease. $T_e$ was arbitrarily taken as the average temperature, 400 K, in the heat exchanger.

streams. Therefore, in a heat exchanger network, the total entropy production rate remains the same, irrespective of how the fluids are paired as long as all inlet and outlet temperatures and heat capacity rates remain the same.

It is to be noted that the irreversible pressure drops in the heat exchanger also generate entropy at a rate of

$$\dot{S}_p = -\oint \frac{\dot{m}v}{T}\,\mathrm{d}p = \int_{p_{ho}}^{p_{hi}} \left(\frac{\dot{m}v}{T}\,\mathrm{d}p\right)_h + \int_{p_{co}}^{p_{ci}} \left(\frac{\dot{m}v}{T}\,\mathrm{d}p\right)_c > 0. \quad (5)$$

Whereas the entropy production rate due to heat transfer, $\dot{S}$, is reflected primarily in the size of the heat exchanger (i.e. in the initial cost of the system), the entropy production rate due to pressure drop, $\dot{S}_p$, affects primarily the pumping power requirements (i.e. the operating cost). In the remainder of this paper, only the former term will be dealt with.

If the entropy production rate is converted into irreversibility rate, $\dot{I}$, by multiplying by an environmental temperature, $T_e$, appropriate for each stream or piece of equipment, we obtain the corresponding power loss. These power losses can be directly compared with each other as well as with the various heat transfer rates or other power terms to guide the design or evaluation of a thermal system.

Returning to equation (1) and Fig. 1, we note that the entropy curves have a direct and unique relation to the load curves used in practice where $T$ is plotted against $q$. The load curves corresponding to Fig. 1 are shown in Fig. 2. A comparison of the two figures demonstrates that the load curves contain, at least qualitatively, all the characteristics of the entropy curves. For example, a smaller net entropy production rate will result in a smaller net area between the curves on both diagrams. Since the load curves show temperature and heat transfer rates directly and since for constant specific heats the curves are straight lines, we shall use only these in the analysis that follows. However, in applications where the magnitudes of the

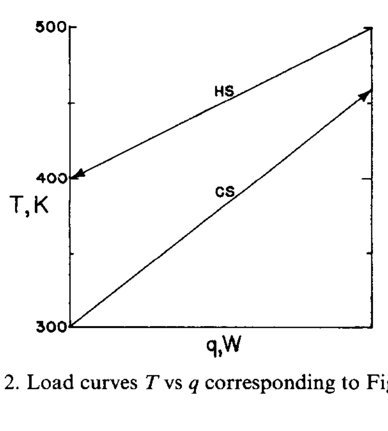


FIG. 2. Load curves $T$ vs $q$ corresponding to Fig. 1.

irreversibilities are sought the entropy curves such as Fig. 1 or equation (4) are needed.

Now if we turn our attention to a heat exchanger network consisting of several heat exchangers in parallel and in series, we can characterize the entire heat transfer process as a single pair of load curves with the hot curve consisting of all the hot stream curves drawn end-to-end along the load axis, $q$, and similarly with the cold curve made up of all the cold stream curves drawn end-to-end. As stated above, the order in which the curves are drawn does not effect the net entropy production rate as long as all flowrates and end temperatures (i.e. all individual loads) are fixed. Consequently, the order must be determined by some other optimizing criterion.

If we take an arbitrary pair of one hot and one cold fluid, the overall heat transfer can be expressed in three ways

$$q_o = C_h(T_{hi} - T_{ho}) \quad (5a)$$

$$q_o = C_c(T_{co} - T_{ci}) \quad (5b)$$

$$q_o = UA\Delta T_{lm}F \simeq UA\overline{\Delta T}F \quad (5c)$$

where the last, approximate expression is actually exact for counterflow heat exchangers if $C_h = C_c$, i.e. when the temperature differences at the two ends are the same, $\Delta T_1/\Delta T_2 = 1$. The error increases slowly with $\Delta T_1/\Delta T_2$. For example, $\overline{\Delta T}$ is 4% higher than $\Delta T_{lm}$ at $\Delta T_1/\Delta T_2 = 2$, and it is 10% higher at $\Delta T_1/\Delta T_2 = 3$. Since the counterflow configuration is optimal for heat exchangers in general, all our analyses will be based on it; thus, $F = 1$.

The heat exchanger conductance, $UA$, needs further examination. Recall the definition of the resistance, $1/UA$

$$\frac{1}{UA} = \frac{1}{h_h A_h \eta_{oh}} + \frac{1}{h_c A_c \eta_{oc}} + R_w \quad (6)$$

where the first two terms on the RHS represent the thermal resistances of the hot and cold fluids, respectively; the overall wall resistance, $R_w$, will be assumed to remain constant. For our purposes then, optimization of a heat exchanger network becomes the minimization of the overall area as expressed in the $UA$ terms. For a given pair of fluids exchanging $q_o$ heat, the greater $\Delta T_{lm}$ (or $\overline{\Delta T}$), the smaller the required $UA$. For two fixed fluids, $UA$ is fixed; however, if there are several hot and cold fluids, the $UA$ values depend on how the fluids are paired. Qualitatively, we can state that the optimum combination of $UA$s will correspond to an arrangement where the combined load curves for both hot and cold sets of fluids increase continually from the cold end towards the hot one with the temperature difference between the two curves kept everywhere as large as possible. The proof is given in the Appendix. This means that the 'coldest' cold fluid should be paired with the 'coldest' hot fluid and the 'hottest' cold fluid with the 'hottest' hot fluid and the others paired similarly in between. From our experience, we suggest that the order be established in terms of increasing outlet temperatures from the cold end towards the hot one for both hot and cold streams. This order however may have to be changed in order to reduce further the total area if the film coefficient, $h$, of a fluid differs considerably from the others. A cold fluid with a low film coefficient may be successfully shifted towards the hot end whereas a cold fluid with a high film coefficient may be moved towards the cold end. Conversely, a hot fluid with a low film coefficient may be displaced towards the cold end whereas a hot fluid with a high film coefficient may be shifted towards the hot end to reduce the total area. When shifting streams, the magnitudes of $q_0$ should be selected such that the number of heat exchangers required is kept as low as possible, thus minimizing the headers and piping required. On the overall load curves, the number of heat exchangers is determined by the number of breaks or discontinuities in both curves, each one of which indicates the start of a new unit. The minimum number of heat exchangers required is equal to the number of either the hot or the cold streams, whichever is greater. Whenever a shift is made in a section of the load curve, its length should be so chosen as to match along the $q$ axis as many of the discontinuities of both load curves as possible, thus reducing the number of heat exchangers required. Equation (A8) must be applied to the original and to the switched configurations to determine which one yields the smaller heat exchanger area.

It is to be noted that this method works equally well even if the load curves are not linear, i.e. with variable heat capacity rates.

## 3. ILLUSTRATIVE EXAMPLES

We will further develop the optimizing methodology by applying it to two typical examples used in [2, 10, 11]. The data are given in Table 1. In order to facilitate comparison with the references, we are also using British units, although the unit specifications are unimportant. As a first step, we can plot the load curves with all the inlet temperatures lined up, i.e. the cold fluids are left adjusted and the hot fluids right adjusted as shown in Figs. 3 and 4. The general optimizing rule is that both hot and cold load curves should be arranged end-to-end starting from the left (or cold) end, in order of increasing outlet temperatures as indicated by the arrowheads. If two cold stream outlet temperatures are the same, then the fluid with the higher heat transfer rate, i.e. with the flatter curve, or with the lower inlet temperature, should be placed closer towards the cold end. If two hot stream outlet temperatures are the same, then the fluid with the higher heat transfer rate, or with the

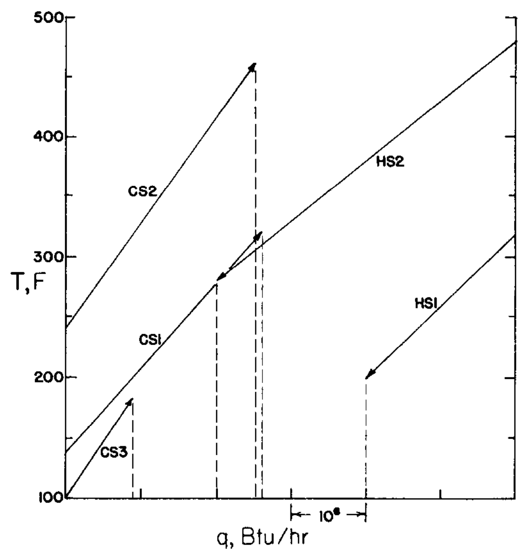


FIG. 3. Lined-up load curves for example 1.

Table 1. Fluid characteristics

| | Stream | Temperatures (°F) Inlet | Outlet | Heat capacity rate (Btu $h^{-1}$ °$F^{-1}$) | Total heat transfer (Btu $h^{-1}$) |
|---|---|---|---|---|---|
| Example 1 | HS1 | 320 | 200 | 16,670 | 2,000,400 |
| | HS2 | 480 | 280 | 20,000 | 4,000,000 |
| | CS1 | 140 | 320 | 14,450 | 2,601,000 |
| | CS2 | 240 | 460 | 11,530 | 2,536,600 |
| | CS3 | 100 | 180 | 10,785 | 862,800 |
| | | | | | $q_T$ = 6,000,400 |
| Example 2 | HS1 | 365 | 265 | 10,000 | 1,000,000 |
| | HS2 | 280 | 220 | 33,333 | 2,000,000 |
| | CS1 | 150 | 250 | 10,000 | 1,000,000 |
| | CS2 | 200 | 300 | 20,000 | 2,000,000 |
| | | | | | $q_T$ = 3,000,000 |

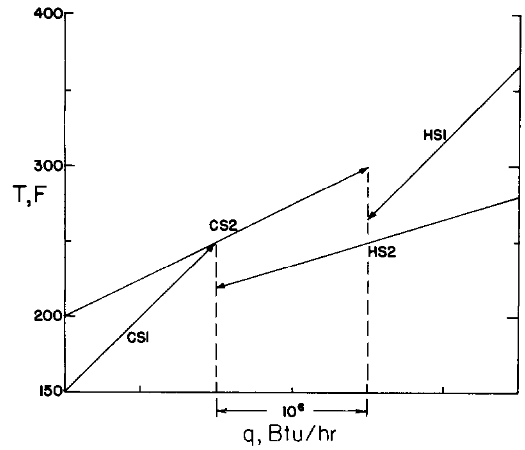


FIG. 4. Lined-up load curves for example 2.

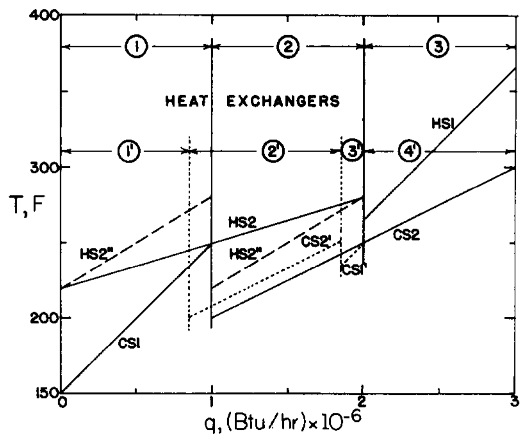


FIG. 6. Overall load curves for example 2 (solid lines show initial layout with 'pinch'; dotted lines show stream switching; dashed lines show stream splitting of HS2, the optimum arrangement).

higher inlet temperature should be placed closer towards the hot end. Since these rules are rather simple, Figs. 3 and 4 are not really necessary but they are helpful to establish the optimum order. Using these rules, we can now establish the overall load curves as shown in Figs. 5 and 6.

Example 1 in Fig. 5 shows an acceptable optimum solution requiring four heat exchangers as determined by the location of the temperature discontinuities and illustrated diagrammatically in Fig. 7.

Example 2 in Fig. 6, however, indicates an unacceptable setup with the outlet end of cold stream 1 (CS1) touching the load curve of hot stream 2 (HS2) and creating a so-called pinch. This would require an infinite size heat exchanger—a physical impossibility. To correct such a situation, when the two curves either touch or even cross (prohibited by the second law of thermodynamics), we may have several alternatives. One is to exchange the 'offending' end of the load curve (CS1) with a corresponding length of the section to the right of it (CS2). The length along the load axis, $q$, to be exchanged should be selected according to the rules described previously. The minimum length in terms of $q$ can be most easily determined by assuming a minimum allowable approach temperature, e.g. 10°F. Another alternative is to limit the maximum heat exchanger effectiveness. In order to keep the number of heat exchangers to a minimum, the high end of CS1 should be matched to the high end of HS2 and the entire lower half of CS2 should be shifted towards the cold end as shown with dotted lines in Fig. 5. Since the breaks in the two load curves were matched in one place, this arrangement needs only one additional heat exchanger.

We have, however, another alternative which requires no additional heat exchangers: namely,

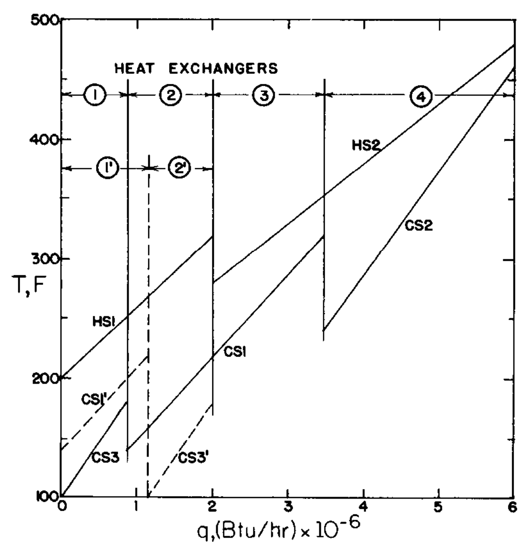


FIG. 5. Overall load curves for example 1 (solid lines show initial optimum arrangement; dashed lines show shifting of CS3 if its film coefficient is very low).

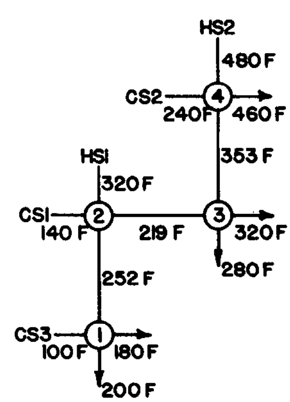


FIG. 7. Diagram of optimum heat exchanger network for example 1.

stream splitting of HS2. This will create two load curves which will have the same overall load length along the $q$ axis as the original single curve but they will have steeper slopes because both will have to have the original temperature drop in order to keep the arrangement optimum. In example 2, the stream splitting can be done to match the end points of CS1 and CS2 as shown with dashed lines in Fig. 5. This is then the optimum, physically realizable arrangement with only three units required and is illustrated diagrammatically in Fig. 8. Both optimum arrangements in Figs. 7 and 8 are identical with those found by computer methods in refs. [2, 10, 11].

Now let us examine the advantages of stream shifting if the film coefficients warrant it. Let us assume that in example 1, Fig. 5, CS3 has a relatively low film coefficient. The load curves indicate that CS3 could be exchanged with the lower part of CS1 without violating the second law. To keep the number of heat exchangers the same, the break point between HS1 and HS2 will be matched with the cold streams as shown with dashed lines in Fig. 5. For the sake of simplicity, assume that the area ratios are unity, the wall resistances are negligible, and that all film coefficients, $h$, and surface effectivenesses, $\eta_o$, are the same except the film coefficient of CS3 which we will designate as $h_{c1}$. Then equation (A8) becomes for the original arrangement

$$\Sigma \equiv (A\eta_o h)_{h1} + (A\eta_o h)_{h2} = \frac{q_1}{\Delta T_1}\left(1+\frac{h}{h_{c1}}\right)+2\frac{q_2}{\Delta T_2} \tag{7}$$

Substituting the values of $q_1 = 862{,}800$ Btu $h^{-1}$, $\Delta T_1 = 85.9$°F, $q_2 = 1{,}137{,}600$ Btu $h^{-1}$, $\Delta T_2 = 106.6$°F yields

$$\Sigma = 10{,}044\left(1+\frac{h}{h_{c1}}\right)+21{,}343 = 31{,}287+10{,}044\frac{h}{h_{c1}} \tag{8}$$

with the switch, the values in equation (7) become $q'_1 = 1{,}137{,}600$ Btu $h^{-1}$, $\Delta T'_1 = 54.7$°F, $q'_2 = 862{,}800$ and $\Delta T'_2 = 154.1$°F, and the modified sum is

$$\begin{aligned}\Sigma' &= \frac{q'_1}{\Delta T'_1}2+\frac{q'_2}{\Delta T'_2}\left(1+\frac{h}{h_{c1}}\right)\\ &= 41{,}594+5599\left(1+\frac{h}{h_{c1}}\right)\\ &= 47{,}193+5599\frac{h}{h_{c1}}.\end{aligned} \tag{9}$$

Comparing equations (8) and (9) reveals that the modified configuration requires smaller heat transfer area and should be used if $h/h_{c1} > 3.56$.

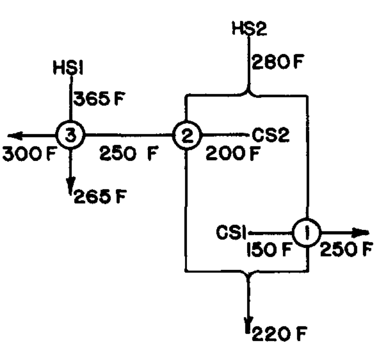


FIG. 8. Diagram of optimum heat exchanger network for example 2.

## 4. CONCLUSIONS AND RECOMMENDATIONS

In contrast to previous computer-based approaches, a simple graphical method has been developed for the optimization of heat exchanger networks with respect to size. It consists of drawing the two (hot and cold) overall load curves, i.e. the individual load curves laid end-to-end, with the outlet temperatures rising continually from the cold end to the hot one and with the temperature difference between the two curves kept everywhere as large as possible. Since the second law of thermodynamics and practical considerations require a minimum distance between the hot and cold load curves, parts of the load curves may have to be shifted or some of the streams may have to be split. Such operations should be accomplished in such a way as to keep the number of heat exchangers to a minimum, i.e. the breaks and discontinuities in the two load curves should be matched along the $q$ axis as much as possible.

The method also allows optimization with respect to variations in the parameters of the heat exchange, particularly the film coefficients of the various fluids. This has not been attempted in the previous computerized solutions.

Although the method could be programmed on a computer, we believe that the simplicity of the method as well as the visual assistance provided by the load curves in suggesting possible alternatives in stream shifting or splitting renders computerization not only unnecessary but restrictive, particularly when the load curves are nonlinear.

We have not attempted to include other economic factors or resiliency, i.e. significant excursions of the fluid parameters, directly into the method. However, its simplicity allows the rather quick examination of different systems and thus provides comparisons to aid the final design.

Although the use of load curves is an established practice, our application to a multi-stream heat exchanger network is new, as can be deduced from the fact that all previous optimization methods depended on search type computer algorithms.

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## APPENDIX

*Proof of the optimizing criterion*

In order to prove that minimization of the $UA$ values corresponds to increasing temperatures along the load curves, consider two arbitrary small sections with the same heat transfer rate, $q$, arranged according to the qualitative optimization rule and find the effect of exchanging one of the fluids, say the cold one, between the two sections as shown in Fig. A1. The original value of the total $UA$ is

$$UA_{\mathrm{T}} = q/\overline{\Delta T}_1 + q/\overline{\Delta T}_2. \tag{A1}$$

The new value with the switch is

$$UA'_1 = \frac{q}{\overline{\Delta T}_1 - \delta_c} + \frac{q}{\overline{\Delta T}_2 + \delta_c} \tag{A2}$$

The ratio can be shown to be

$$\frac{UA_{\mathrm{T}}}{UA'_{\mathrm{T}}} = \frac{(\overline{\Delta T}_1 - \delta_c)(\overline{\Delta T}_2 + \delta_c)}{\overline{\Delta T}_1\,\overline{\Delta T}_2}$$

$$= 1 - \frac{\delta_c}{\overline{\Delta T}_1\,\overline{\Delta T}_2}[\delta_c - (\overline{\Delta T}_1 - \overline{\Delta T}_2)] \tag{A3}$$

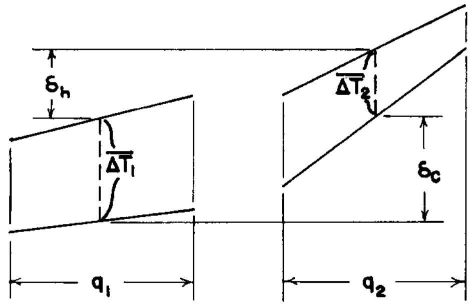


FIG. A1. Load curve diagram for stream switching.

but

$$\overline{\Delta T}_1 - \overline{\Delta T}_2 = \delta_c - \delta_h \tag{A4}$$

therefore

$$\frac{UA_{\mathrm{T}}}{UA'_{\mathrm{T}}} = 1 - \frac{\delta_h\,\delta_c}{\overline{\Delta T}_1\,\overline{\Delta T}_2}. \tag{A5}$$

The result is the same if the hot fluids are switched. Thus, as long as both $\delta_h$ and $\delta_c$ are positive (a consequence of the optimization criterion used here), the original arrangement yields a smaller, therefore generally better, $UA$. If either $\delta$ is zero, then the two arrangements give the same $UA$.

The optimizing criterion given in the text, using the exit temperatures, is a simplified extension of this proof from equal heat transfer rates to unequal ones. It works better than using the average stream temperatures.

*Derivation of the switch criterion*

Refer to Fig. A1 and consider the effects of the film coefficients when switching one of the fluids, say the cold one. The sections in this case do not have to transfer the same amount of heat and their values will be switched also. The hot fluid curves will also have to be matched. Express $UA$ in terms of equations (5c) and (6) for the original arrangement

$$\frac{1}{(A\eta_o)_{h1}}\frac{1}{h_{h1}} + \frac{(A\eta_o)_{h1}}{h_{c1}(A\eta_o)_{c1}} + (A\eta_o)_{h1}R_{w1} = \frac{\overline{\Delta T}_1}{q_1} \tag{A6}$$

$$\frac{1}{(A\eta_o)_{h2}}\frac{1}{h_{h2}} + \frac{(A\eta_o)_{h2}}{h_{c2}(A\eta_o)_{c2}} + (A\eta_o)_{h2}R_{w2} = \frac{\overline{\Delta T}_2}{q_2}. \tag{A7}$$

The total hot side area can be expressed as

$$(A\eta_o)_{h1} + (A\eta_o)_{h2} = \left[\left(\frac{1}{h_{h1}} + \frac{(A\eta_o)_{h1}}{h_{c1}(A\eta_o)_{c1}}\right)\Big/\left(\frac{\overline{\Delta T}_1}{q_1} - R_{w1}\right)\right] + \left[\left(\frac{1}{h_{h2}} + \frac{(A\eta_o)_{h2}}{h_{c2}(A\eta_o)_{c2}}\right)\Big/\left(\frac{\overline{\Delta T}_2}{q_2} - R_{w2}\right)\right]. \tag{A8}$$

After switching the cold fluids, the total hot side area can be found from equation (A8) with the appropriate values of the switched variables (e.g. $h'_{c1}$ and $h'_{c2}$ or $q'_1$ and $q'_2$). A similar expression can be easily derived for the total cold side area which may be preferable to use if the hot fluids are switched. If a switch requires the change in more than two heat exchanger sections, then equations similar to equations (A6) or (A7) have to be developed for each section and the corresponding areas (either hot or cold side) have to be summed to obtain a total area similar to equation (A8).

## OPTIMISATION DES RESEAUX D'ECHANGEURS DE CHALEUR BASEE SUR LA SECONDE LOI ET DES COURBES DE CHARGE

**Résumé**—Le problème de l'optimisation simultanée des échangeurs de chaleur pour un nombre donné de fluides chauds et froids est approché à partir de la seconde loi de la thermodynamique. Si les conditions des besoins sont fixées, par exemple toutes les températures d'entrée et de sortie sont données pour les fluides, alors la production d'entropie est indépendante des couples de fluides. Et l'optimisation devient essentiellement une maximalisation des différences de température entre les fluides couplés pour tous les fluides considérés. On montre que la même optimisation peut être obtenue à partir des courbes de charge où la température est portée en fonction du transfert de chaleur pour chacun des fluides. Si les coefficients de film pour les fluides sont significativement différents les uns des autres, on peut donner un critère pour déterminer l'optimisation. La méthode est extrêmement simple et peut être conduite avec efficacité sur un papier à dessin.

## OPTIMIERUNG VON WÄRMEÜBERTRAGERNETWERKEN AUFGRUND DES 2. HAUPTSATZES UNTER VERWENDUNG VON BELASTUNGSKURVEN

**Zusammenfassung**—Das Problem der simultanen Optimierung von Wärmetauschern für mehrere heiße und kalte Fluide wird unter dem Gesichtspunkt des 2. Hauptsatzes der Thermodynamik angegangen. Wenn das Lastenheft feststeht, d. h. alle Vor- und Rücklauftemperaturen der Fluide sind festgelegt, ist die Entropie-Erzeugung von den tatsächlichen Fluidpaarungen unabhängig. Folglich geht eine Optimierung in Bezug auf die Größe im Grunde genommen in eine Maximierung der Temperaturunterschiede zwischen sämtlichen Fluidpaaren über. Weiterhin wird gezeigt, daß dieselbe Optimierung aus den Belastungskurven ermittelt werden kann, bei denen für jedes Fluid die Temperatur über der ausgetauschten Wärme aufgetragen wird. Wenn der Wärmeübergangskoeffizient der einzelnen Fluide verschieden von dem der anderen ist, kann eine einfache Verlagerung erreicht werden, und es folgt ein Kriterium, das angibt, ob eine solche Verlagerung die Optimierung verbessert. Die Methode ist sehr einfach und kann höchst effektiv grafisch durchgeführt werden.

## ОПТИМИЗАЦИЯ СИСТЕМЫ ТЕПЛООБМЕННИКОВ, ОСНОВАННАЯ НА ВТОРОМ ЗАКОНЕ ТЕРМОДИНАМИКИ, С ИСПОЛЬЗОВАНИЕМ КРИВЫХ НАГРУЗКИ

**Аннотация**—Задача единой оптимизации теплообменников для ряда горячих и холодных жидкостей рассматривается с точки зрения второго закона термодинамики. В случае, когда заданы требования на общую мощность, т.е. установлены все входные и выходные температуры жидкостей, результирующая скорость производства энтропии не зависит от реальных пар жидкостей. Таким образом, оптимизация с учетом размера сводится к максимизации перепадов температур всех рассматриваемых пар жидкостей. Такая же оптимизация может быть получена из кривых нагрузки, в которых график температуры строится относительно теплопередачи для каждой из жидкостей. В случае, когда коэффициент пленочной теплопередачи для какой либо жидкости значительно отличается от коэффициентов других жидкостей, можно выполнить простое перемещение графика и дать критерий, определяющий, способствует ли такое перемещение оптимизации. Метод очень прост и может быть эффективно реализован с помощью миллиметровой бумаги.